Electrostatic Sensing & Actuation

(Source: Chapter 4, Liu & Change)

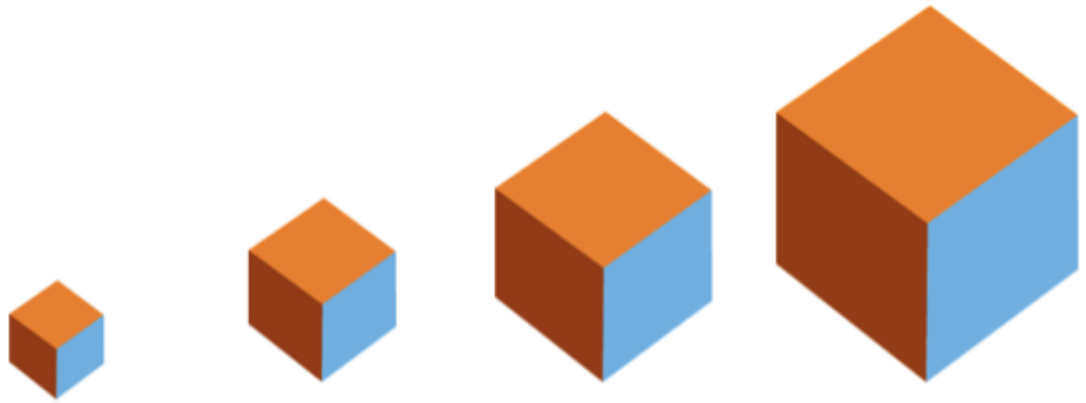
Electrostatic Sensing & Actuation

A capacitor is broadly defined as two conductors that can hold opposite charges which can be used as either a sensor or an actuator.

If the distance and relative position between two conductors change as a result of applied stimulus, the capacitance value would be changed. This forms the basis of capacitive (or electrostatic) sensing.

If a voltage (or electric field) were applied across two conductors, an electrostatic force would develop between these two objects. This is defined as electrostatic actuation.

Electrostatic based devices are attractive candidate for micro actuation??

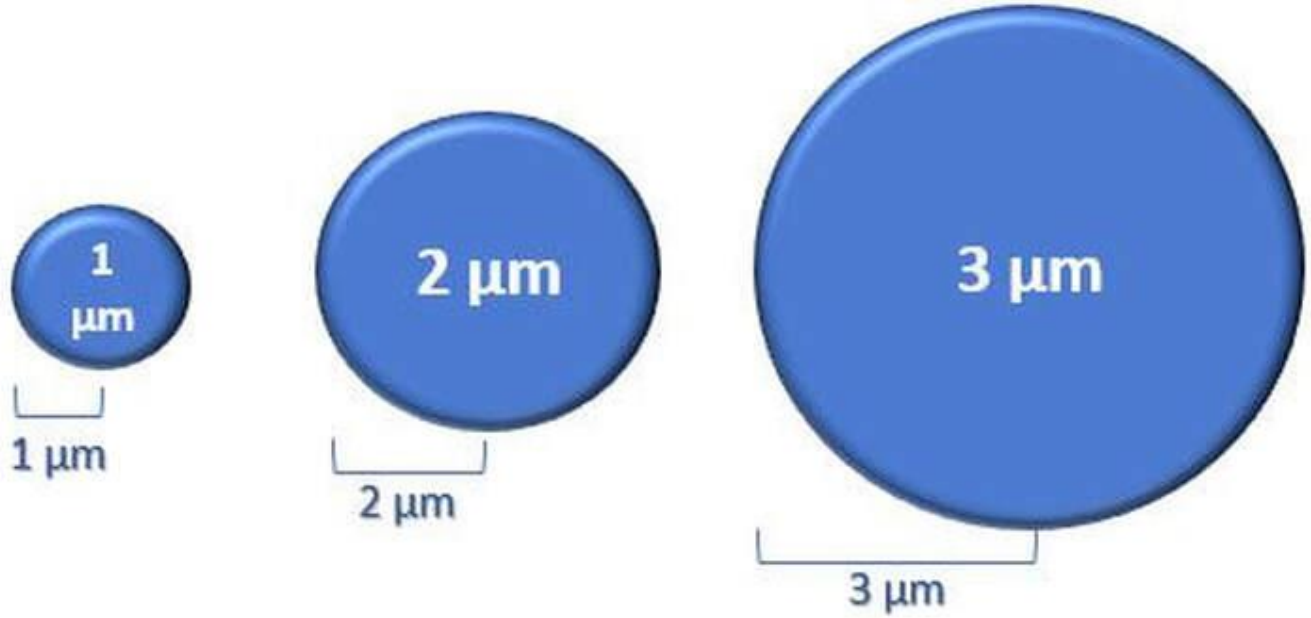


Surface Area (cm^2)	24	96	216	384
Volume (cm^3)	8	64	216	512
Surface area / volume	3	1.5	1	0.75

Fig 1. Surface Area to Volume Ratio. As an object increases in size, its surface area to volume ratio decreases.

Electrostatic based devices are attractive candidate for micro actuation??

Micro devices have large surface-area-to-volume ratios and their masses are generally very small, thus making electrostatic force, which is a *surface force*, an attractive candidate for micro actuation.



$Diameter_{(sphere)}$	1 µm	2 µm	3 µm
$Surface\ Area_{(sphere)} = 4\pi r^2$	3.14 μm^2	12.56 μm^2	28.26 μm^2
$Volume_{(sphere)} = \frac{4\pi r^3}{3}$	0.52 μm^3	4.19 μm^3	14.18 μm^3
Surface Area-to-Volume Ratio	**6:1**	**3:1**	**2:1**

Voltage: ~100 volts to overcome static friction
Torque: ~Pico N-m

Micro-Motor

What are the typical voltage & torque values?

- The electrostatically driven micro motor consists of a rotor that is attached to the substrate with a hub, and a set of fixed electrodes on the periphery, called stators.
- The stators are grouped together such that each group of four electrodes is electrically biased simultaneously. Three such groups are present for the motor shown and are identified by different fill patterns.

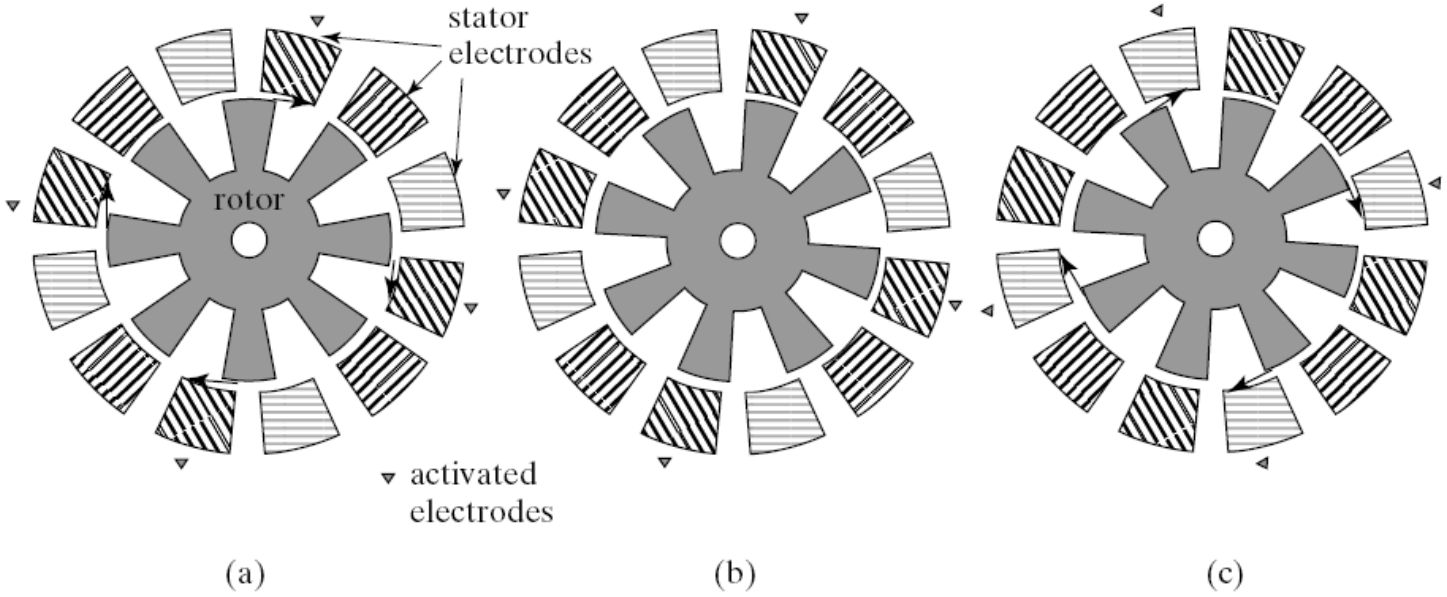


FIGURE 4.1

An electrostatic micro motor.

Major advantages of electrostatic sensing and actuation

Simplicity:

- The sensing and actuation principles are relatively easy to implement, requiring only two conducting surfaces. No special functional materials are required. Other sensing methods, such as piezoresistive and piezoelectric sensing, and other actuation methods, such as piezoelectric actuation, require deposition, patterning, and integration of special piezoresistive and piezoelectric materials.

Low power:

- Electrostatic actuation relies on differential voltage rather than current. The method is generally considered energy efficient for low-frequency applications. This is especially true under static conditions, when no current is involved. At high frequencies, a time- and frequency dependent displacement current, $i(t)$, will develop in response to a time-varying bias voltage $V(t)$.

Fast response:

- Electrostatic sensing and actuation offers high dynamic response speed, as the transition speed is governed by the charging and discharging time constants that are typically small for good conductors. For example, the switching time of the mirrors in the digital micromirror display (DMD) array is smaller than fast enough to support 8-bit gray scale display.

Issues with electrostatic sensing and actuation

High voltage required for static actuator operation is considered a drawback. The DMD mirrors are switched by voltage on the order of 25 V to achieve ± 7.5 degree tilting.

A monolithically integrated optical mirror with 9° tilt range requires 150 V bias voltage.

Linear electrostatic actuators requiring hundreds of volts to move a microstructure by several tens of micrometers are rather common.

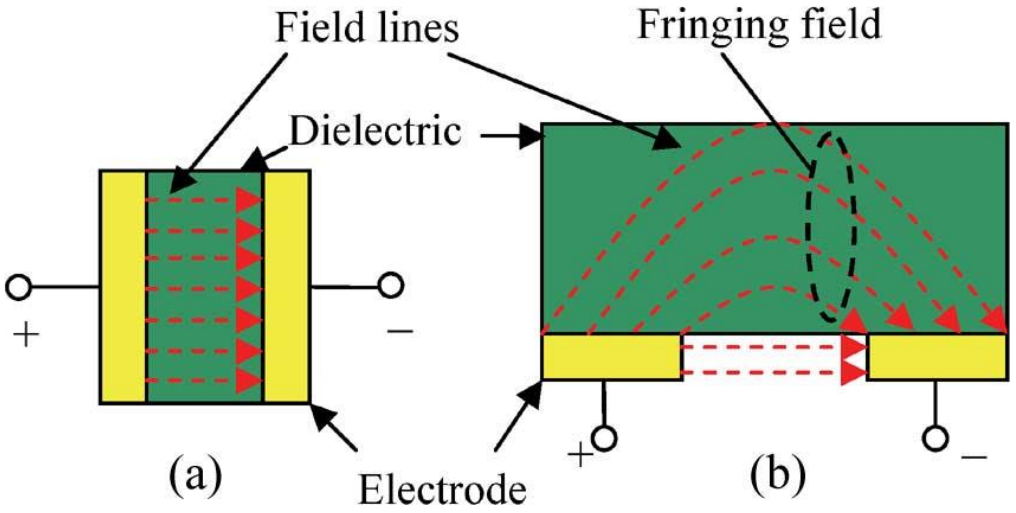
High voltage introduces electronics complexity (for providing high voltage supply) and material compatibility issues.

Types of Capacitive Geometries

For capacitive sensors, careful design of electrical circuits is needed to register capacitance changes on the order of pF or lower, in the presence of noise and interference sources.

There are two major categories of capacitive electrode geometries: parallel-plate capacitor and *interdigitated fringing (comb-drive) capacitors.

The static capacitance values of parallel plates and comb drives are generally very small, being on the order of pF or smaller.



*Ref: Power cable insulation damage detection

Types of Capacitive Geometries

- Capacitive actuators often take advantage of electrostatic *attraction* between two oppositely charged surfaces. Repulsive electrostatic forces, which are used less often, can be developed between two surfaces with same charge polarities.
- For example, vertical levitation of interdigitated fingers up to 0.5 μm has been demonstrated under applied voltage of 20 V for an 18-finger device.

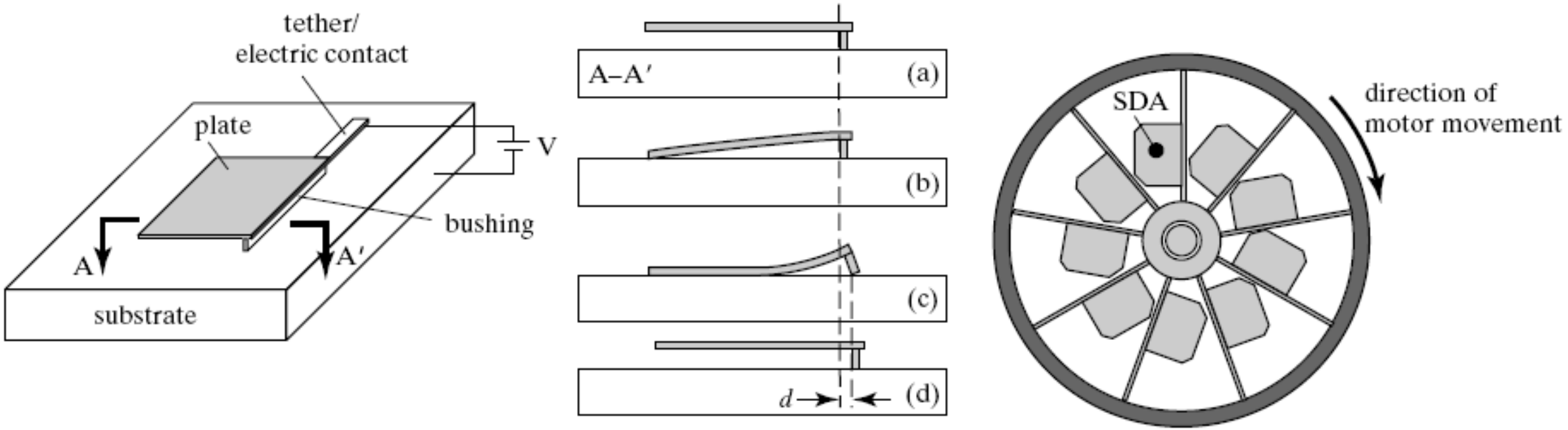


FIGURE 4.15

The operation principle of scratch drive actuators.

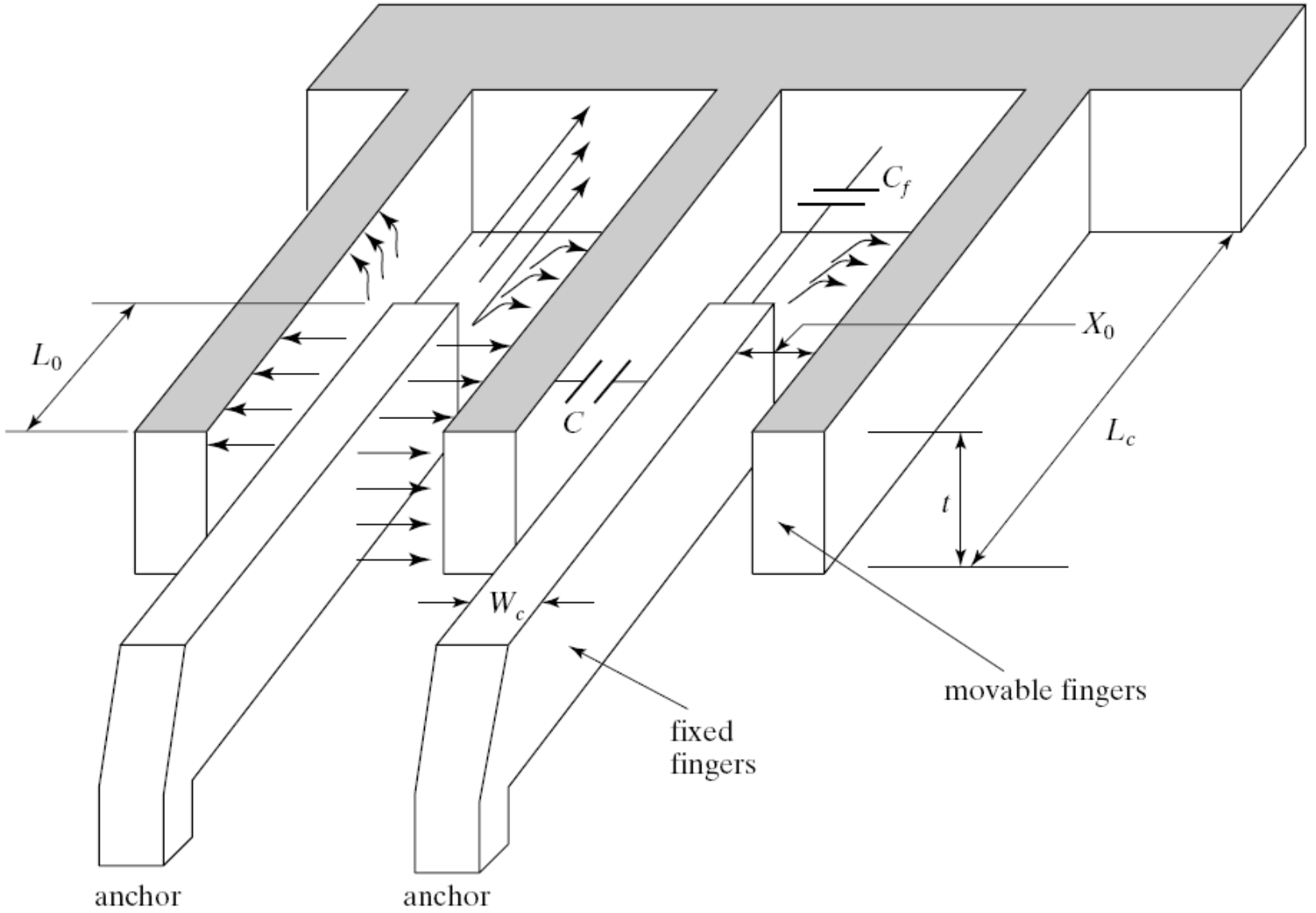


FIGURE 4.16

Perspective view of comb-drive sensors and actuators.

Parallel Plate Capacitor

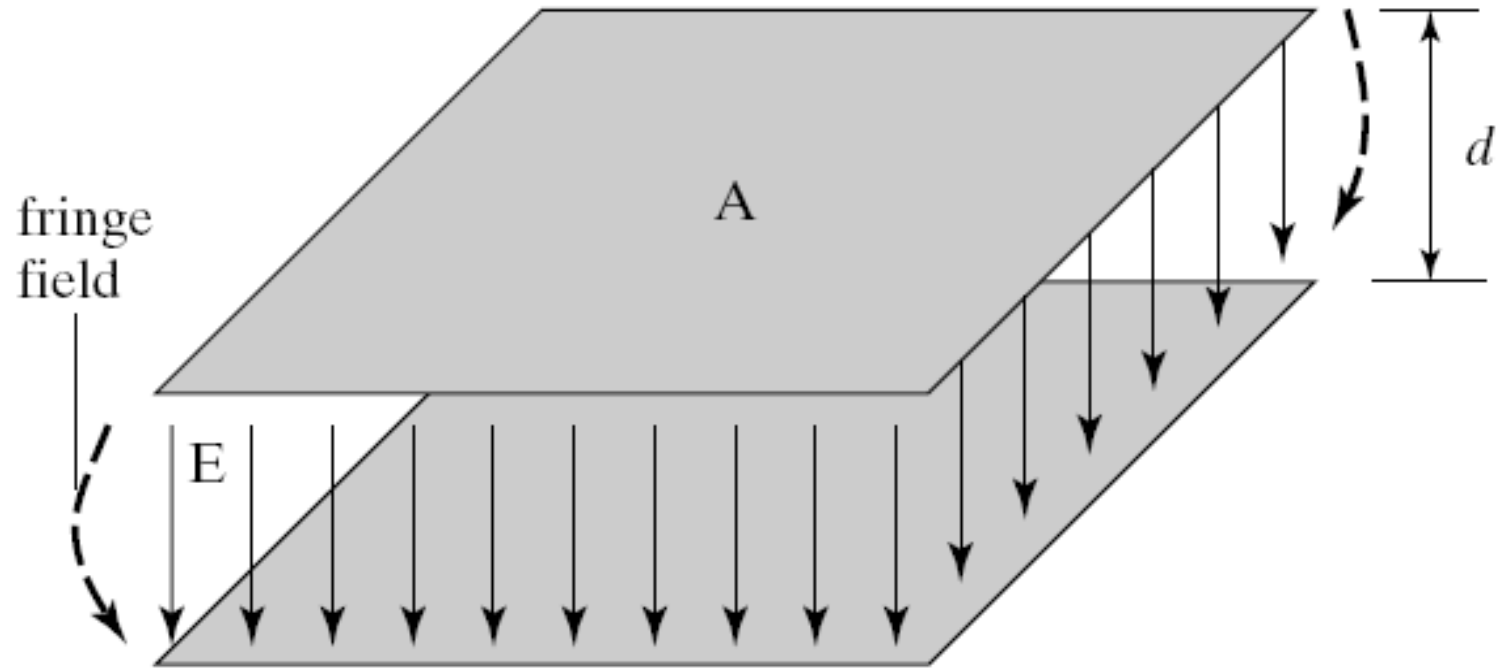


FIGURE 4.2

A parallel-plate capacitor with overlapping area of A and a spacing of d.

$$C \equiv \frac{Q}{V} = \frac{Q}{E \cdot d} = \frac{Q}{\frac{Q}{\varepsilon A} d} = \frac{\varepsilon A}{d}.$$

$$F = \left| \frac{\partial U}{\partial d} \right| = \frac{1}{2} \frac{\varepsilon A}{d^2} V^2 = \frac{1}{2} \frac{CV^2}{d}$$

Example Problem

Consider an air-gap capacitor made with two fixed parallel planar plates. At rest (zero bias), the distance between two parallel plates is $x_0 = 100\,\mu$m, and the areas of plates are $A = 400 \times 400\,\mu\text{m}^2$. The media between the two plates is air. The biasing voltage between these two plates is $V = 5$ Volts. Calculate the numerical value of the capacitance and the magnitude of the attractive force (F). What is the capacitance value if half of the area is filled with water (as the inter plate media)?

Solution. To find the capacitance value, we use

$$C = \varepsilon_r \varepsilon_0 \frac{A}{x_0} = 14.17 \times 10^{-15}\,\text{F}.$$

The force can be calculated using

$$F = \frac{\partial E}{\partial d} = \frac{1}{2}\frac{CV^2}{x_0} = 1.8 \times 10^{-9}\,\text{N}.$$

Example Problem

If one half of the plate area is occupied with water and another half by air, each half can be considered separately. The total capacitance consists of two capacitors connected in parallel. The relative dielectric constant of water at room temperature is 76.6. The capacitance associated with the half with water media is

$$C_{\text{water}} = \varepsilon_r \varepsilon_0 \frac{A/2}{x_0} = \frac{76.6 \times 8.854 \times 10^{-12} \times 80000 \times 10^{-12}}{100 \times 10^{-6}} = 542.6 \times 10^{-15}\ \text{F}.$$

The capacitance of the half with air media is $C_{\text{air}} = 7.08 \times 10^{-15}$ F

The total capacitance is

$$C = C_{\text{air}} + C_{\text{water}} = 549.6 \times 10^{-15}\ \text{F}.$$

What about Force Value?

Equilibrium Position of Electrostatic Actuator Under Bias

- Many electrostatic sensors and actuators involve at least one deformable plate supported by springs.
- An important design aspect for such sensors and actuators is to determine the amount of static displacement under a certain biased voltage.

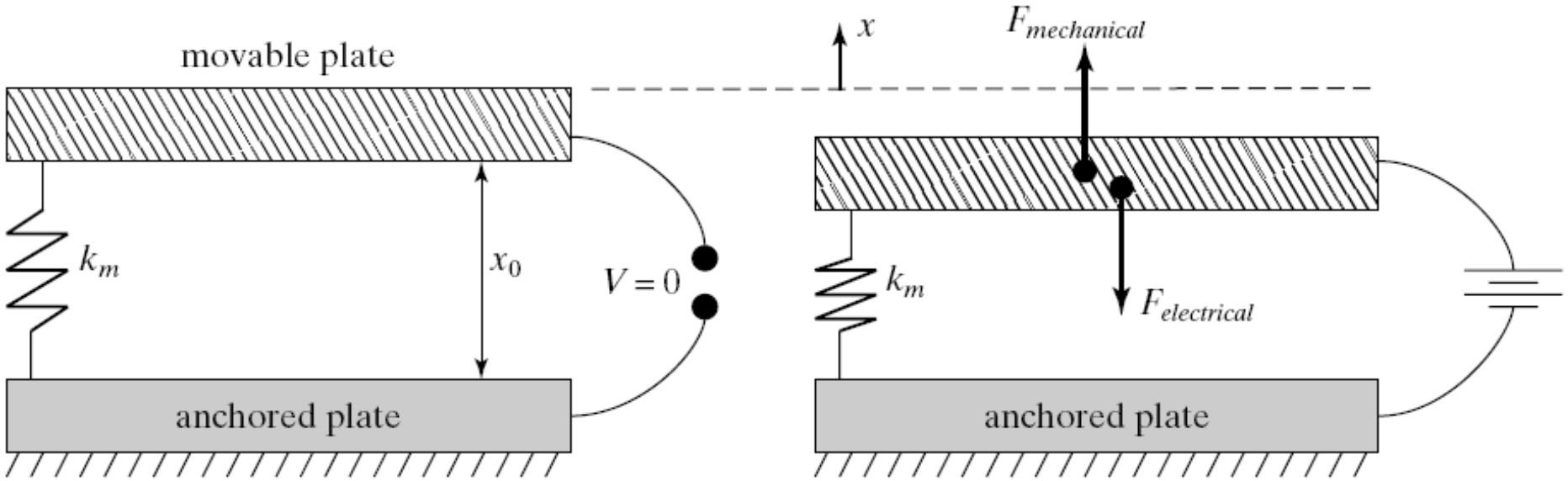


FIGURE 4.3

A coupled electro-mechanical model.

Equilibrium Position of Electrostatic Actuator Under Bias

A parallel-plate capacitor with one movable plate supported by a mechanical spring is diagramed in Figure 4.3. The top plate is supported by a spring with the force constant being K_m. At rest, the applied voltage, displacement, and the mechanical restoring forces are zero. Gravity does not play an important role in static analysis of micro devices, because the mass of plates is generally very small and the gravitational force would not cause appreciable static displacement.

When a voltage is applied, an electrostatic force F_{electric} will be developed. The magnitude of F_{electric} when the movable plate is at its starting position is given by

$$F_{\text{electric}} = \frac{1}{2}\frac{\varepsilon A}{d^2}V^2 = \frac{1}{2}\frac{CV^2}{d}. \tag{4.7}$$

This force will tend to decrease the gap, which gives rise to displacements and mechanical restoring force. Under static equilibrium, the mechanical restoring force has equal magnitude but opposite direction as the electrostatic force.

Equilibrium Position of Electrostatic Actuator Under Bias

The effective spring constant of a structure is the mechanical spring constant minus the electrical spring constant.

Let us derive the equilibrium displacement of a spring-supported electrode plate under a bias voltage V. Suppose the resulted equilibrium displacement is x, with the x-axis pointing in the direction of increasing gap (Figure 4.3). With the displacement x, the gap between electrodes becomes $d + x$ ($d + x < d$). The electrostatic force at the equilibrium position is

$$F_{\text{electric}} = \frac{1}{2}\frac{\varepsilon A}{(x_0 + x)^2}V^2 = \frac{1}{2}\frac{C(x)V^2}{(x_0 + x)}. \tag{4.9}$$

The magnitude of the mechanical restoring force is

$$F_{\text{mechanical}} = -k_m x. \tag{4.10}$$

Equating the magnitudes of $F_{\text{mechanical}}$ and $F_{\text{mechanical,}}$ at x and rearranging terms, we obtain

$$-x = \frac{F_{\text{mechanical}}}{K_m} = \frac{F_{\text{electrical}}}{K_m} = \frac{C(x)V^2}{2(x_0 + x)K_m}. \tag{4.11}$$

The equilibrium distance between two capacitor plates can be calculated by solving the quadratic equation above with respect to x.

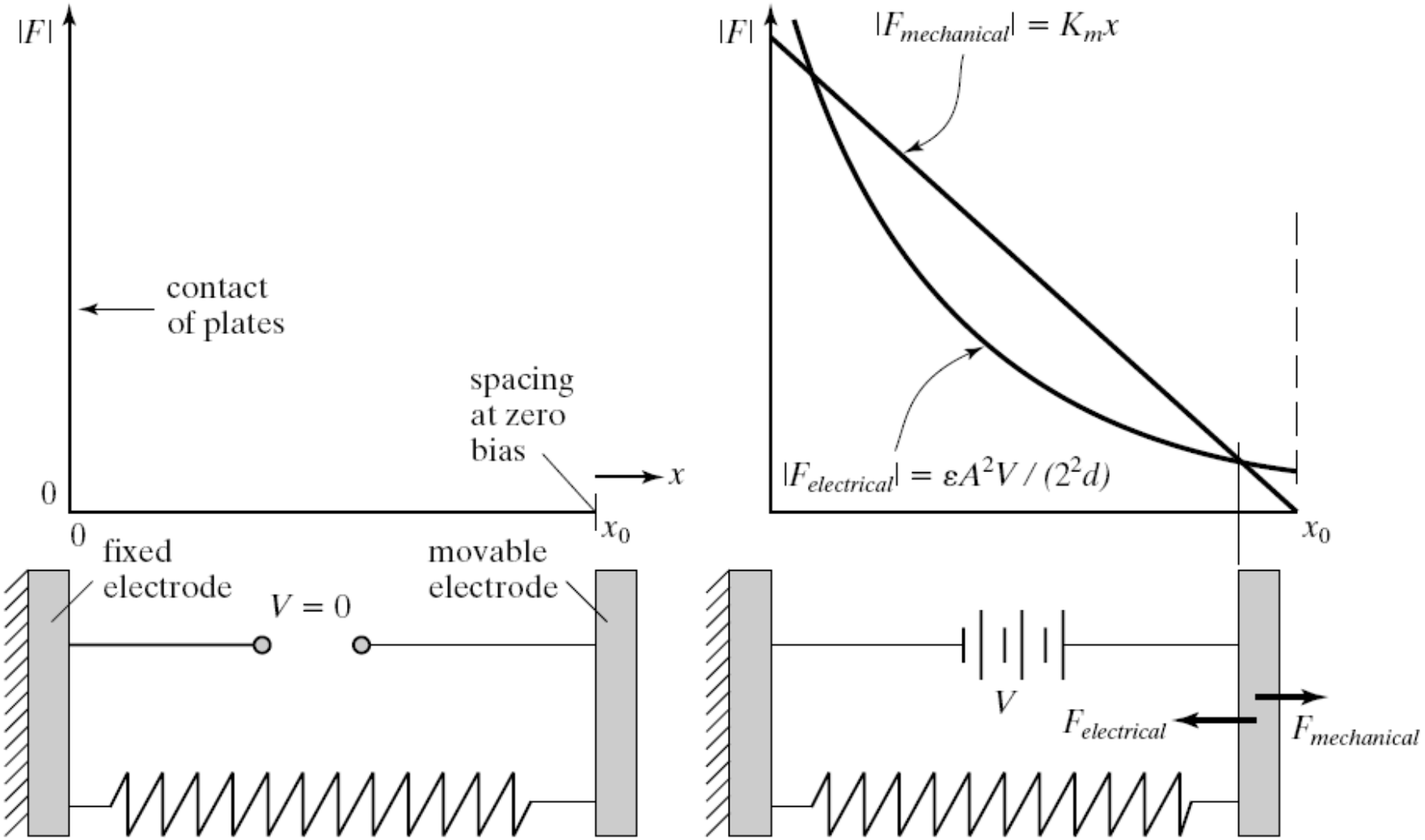


FIGURE 4.4

The magnitude of the electric force and mechanical force as a function of plate spacing.

Electric & Mechanical Force

Electric & Mechanical Force for changing stimuli

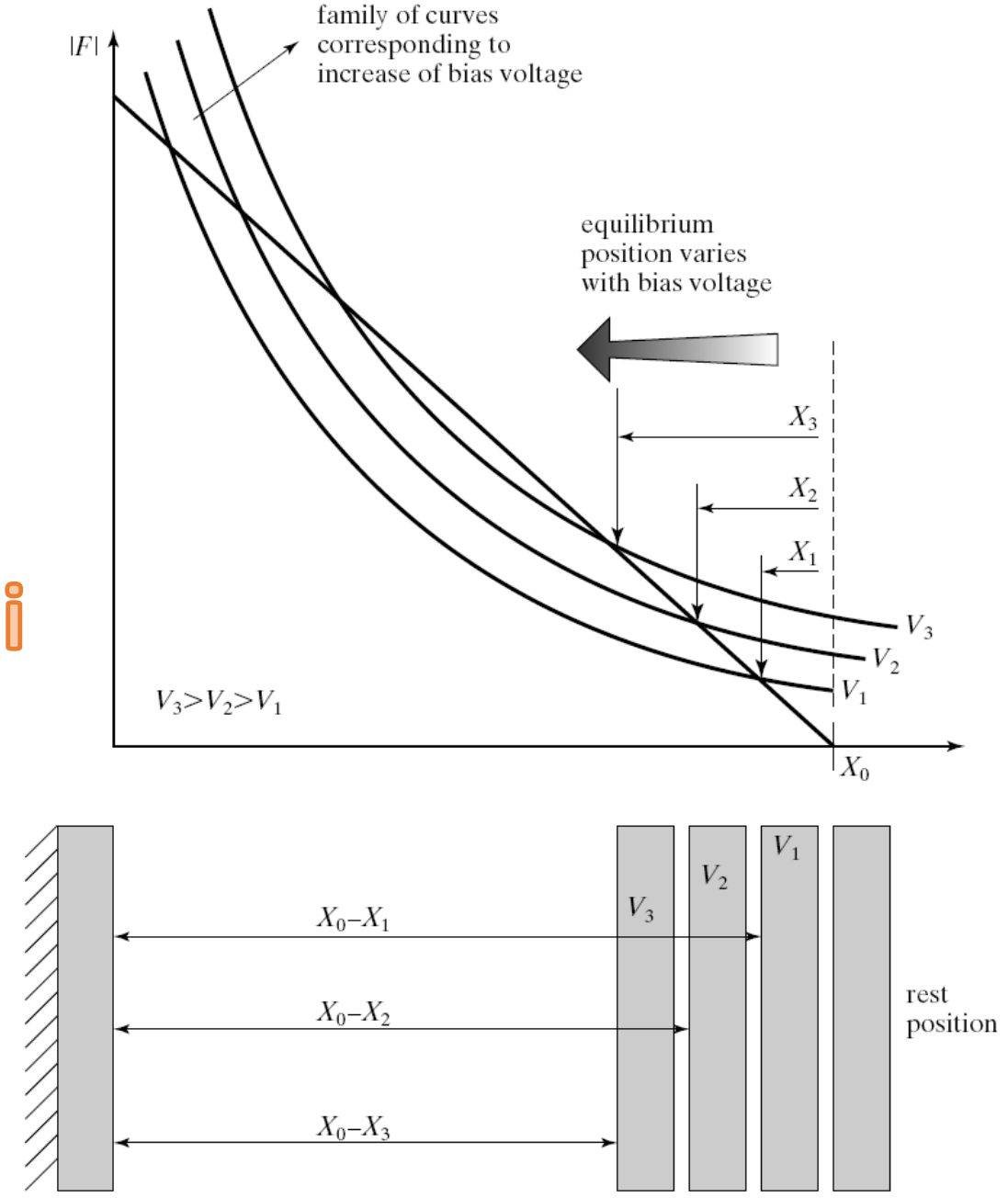


FIGURE 4.5

Balance of electrical and mechanical forces under three bias voltages.

Example 4.2 Calculation of Equilibrium Position

A parallel-plate capacitor is suspended by two fixed-guided cantilever beams, each with length, width and thickness denoted l, w and t, respectively (Figure 4.7). The material is polysilicon, with a Young's modulus of 120 GPa. ($l = 400\,\mu$m, $w = 10\,\mu$m, and $t = 1\,\mu$m.) The gap x_0 between two plates is 2 μm. The area of the plate is 400 μm by 400 μm.

Calculate the amount of vertical displacement when a voltage of 0.4 volts is applied. Repeat the calculation of displacement for the voltage of 0.2 volts.

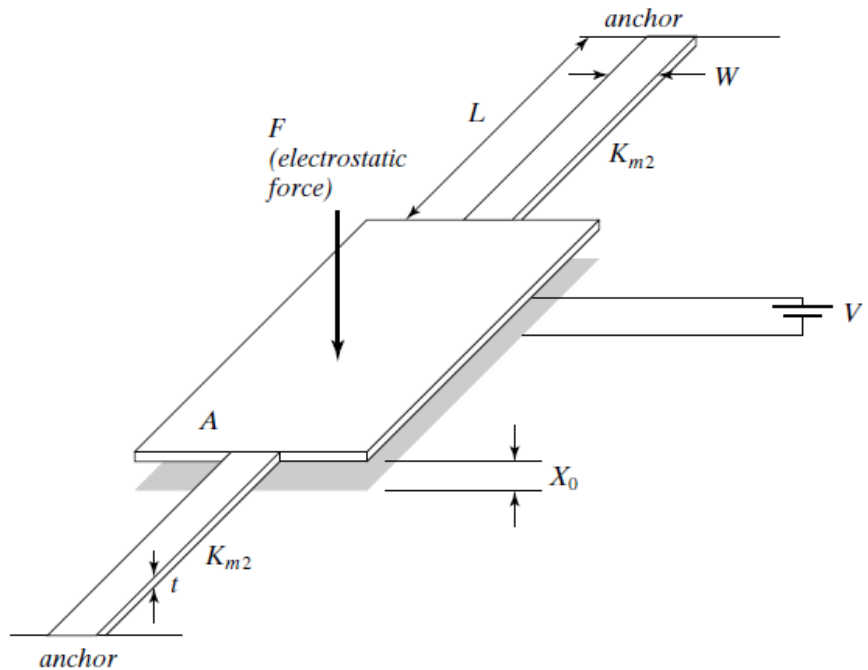


FIGURE 4.7

An electrostatic actuator plate supported by two fixed-guided beams.

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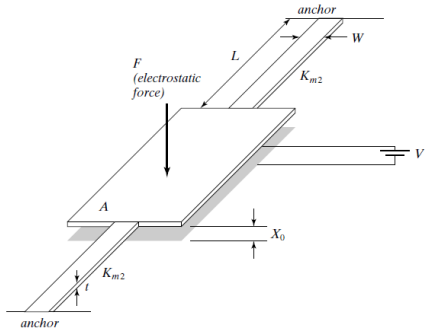


FIGURE 4.7
An electrostatic actuator plate supported by two fixed-guided beams.

Solution. There are three steps in a standard analysis procedure.

Step 1. Find out the force constant associated with the actuator.

We calculate force constant of one beam first using a model of a fixed-guided beam. For each beam, the vertical displacement under applied force F is

$$d = \frac{Fl^3}{12EI}.$$

$$I = \frac{wt^3}{12}$$

$$d = \frac{Fl^3}{12EI}.$$

The force constant is therefore

$$K_m = \frac{F}{d} = \frac{12EI}{l^3} = \frac{Ewt^3}{l^3} = \frac{120 \times 10^9 \times 10 \times 10^{-6} \times (1 \times 10^{-6})^3}{(400 \times 10^{-6})^3} = 0.01875\ \text{N/m}.$$

$$K_m = \frac{F}{d}$$

Because the plate is supported by two beams that are connected in parallel, the overall force constant is the sum of two separate force constants, namely

$$K_m = 0.0375\ \text{N/m}.$$

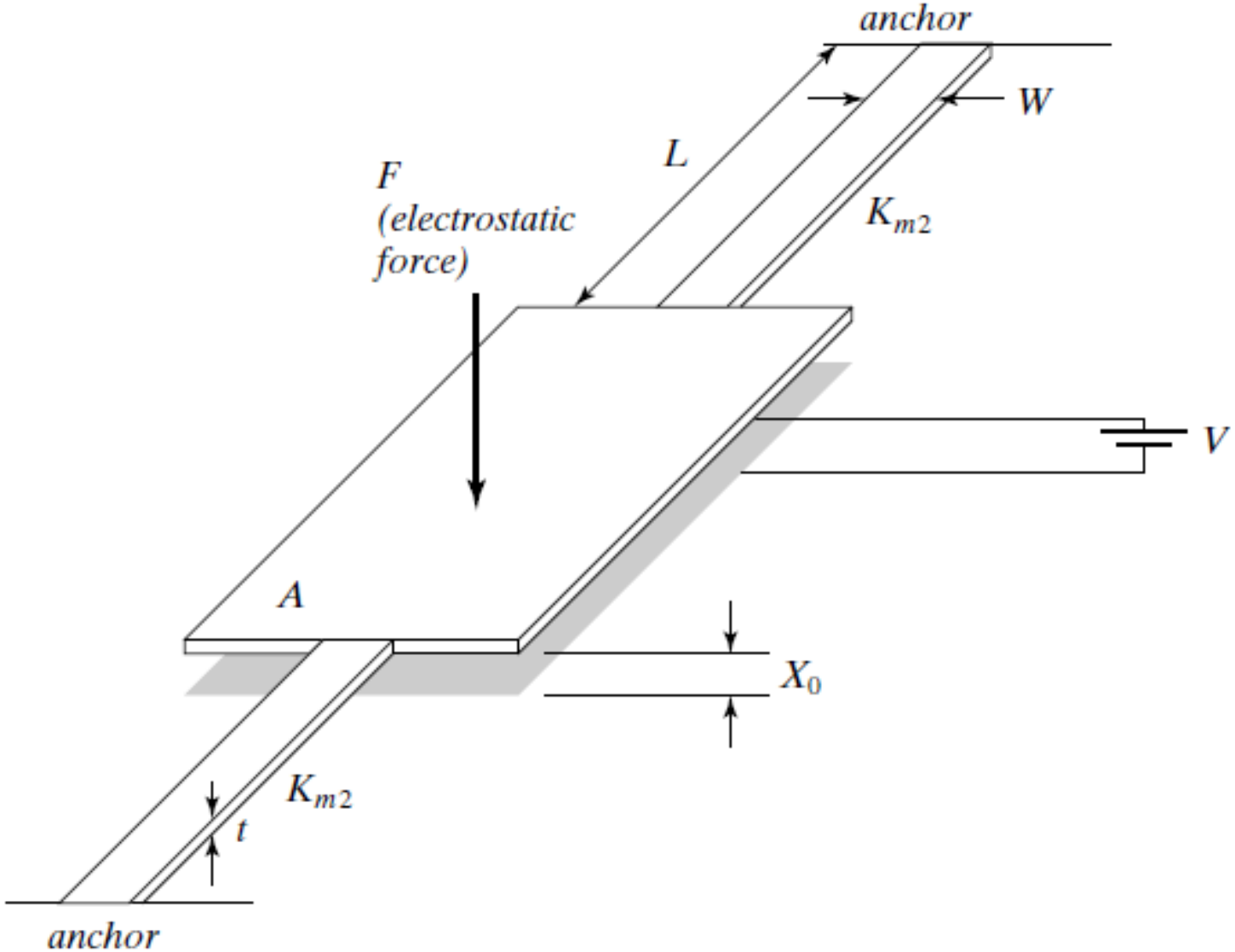
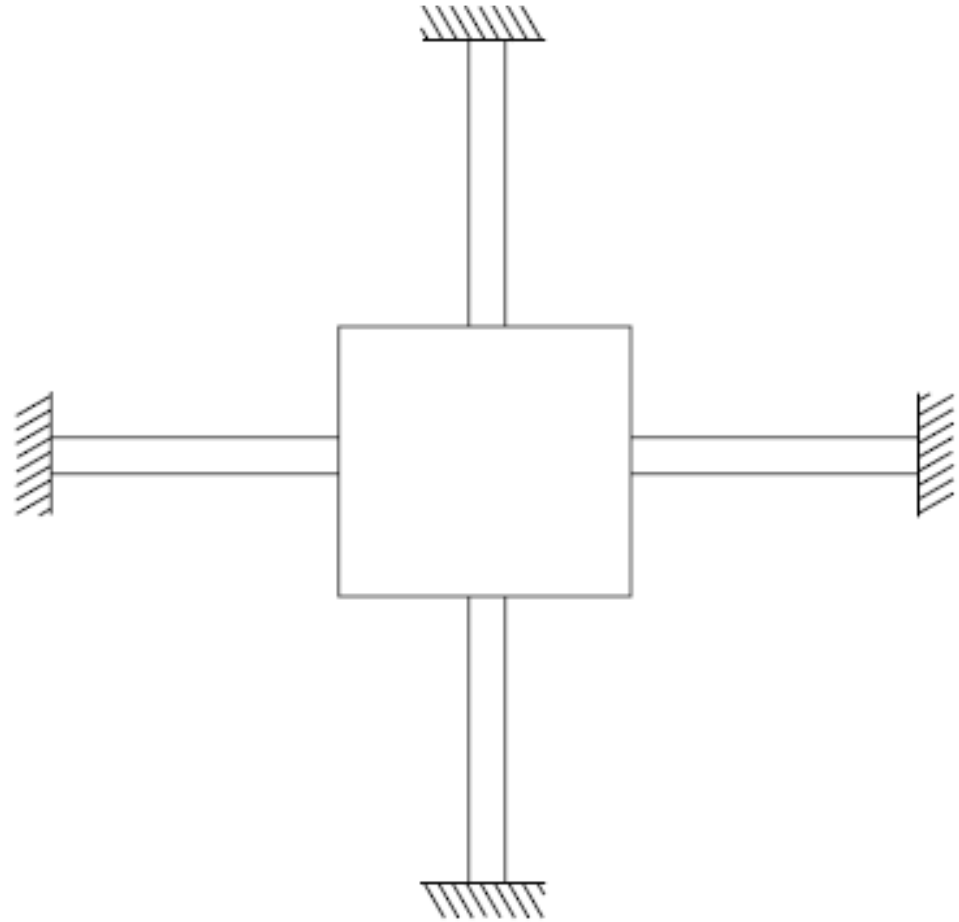


FIGURE 4.7

An electrostatic actuator plate supported by two fixed-guided beams.

Example 4.3 Capacitance Sensor Response

A parallel capacitor with an area (A) of $100 \times 100\ \mu m^2$ is supported by four cantilever beams. The plate is made of polycrystalline silicon that is $t = 2\ \mu m$ thick. The distance between the bottom of the plate and the substrate is $d = 1\ \mu m$. Each cantilever beam is $400\ \mu m$ long (l), $20\ \mu m$ wide (w), and $0.1\ \mu m$ thick (t). Find the relative change of capacitance under an acceleration of 1 g.



Solution. The mass of the plate is

$$m = \rho A T = 2330 \times (100 \times 10^{-6})^2 \times 2 \times 10^{-6} = 46.6 \times 10^{-12}\ \text{kg}.$$

The magnitude of the force acting on the plate under an acceleration a is

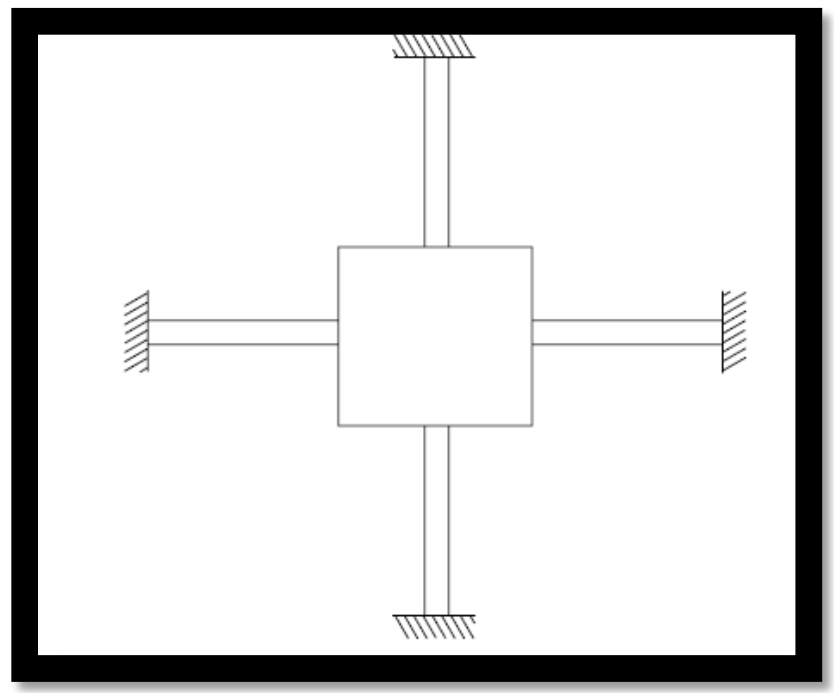
$$F = ma = 46.6 \times 10^{-12}\ \text{N}.$$

Suppose the Young's modulus of polysilicon is 150 GPa. The force constant associated with all four support beams (fixed-guided) is

$$k = 4 \times \left(\frac{12EI}{l^3}\right) = \frac{48 \times 150 \times 10^9 \times \dfrac{20 \times 10^{-6} \times (0.1 \times 10^{-6})^3}{12}}{(400 \times 10^{-6})^3} = 0.0001875\ \text{N/m}.$$

The static displacement under the applied acceleration is

$$\delta = \frac{F}{k} = 0.248\ \mu\text{m}.$$



The capacitance value under zero applied force is

$$C_0 = \varepsilon \frac{A}{d} = \frac{8.85 \times 10^{-12} \times (100 \times 10^{-6})^2}{1 \times 10^{-6}} = 88.5 \times 10^{-15}\ \text{F}.$$

The value of the capacitance after the displacement δ is

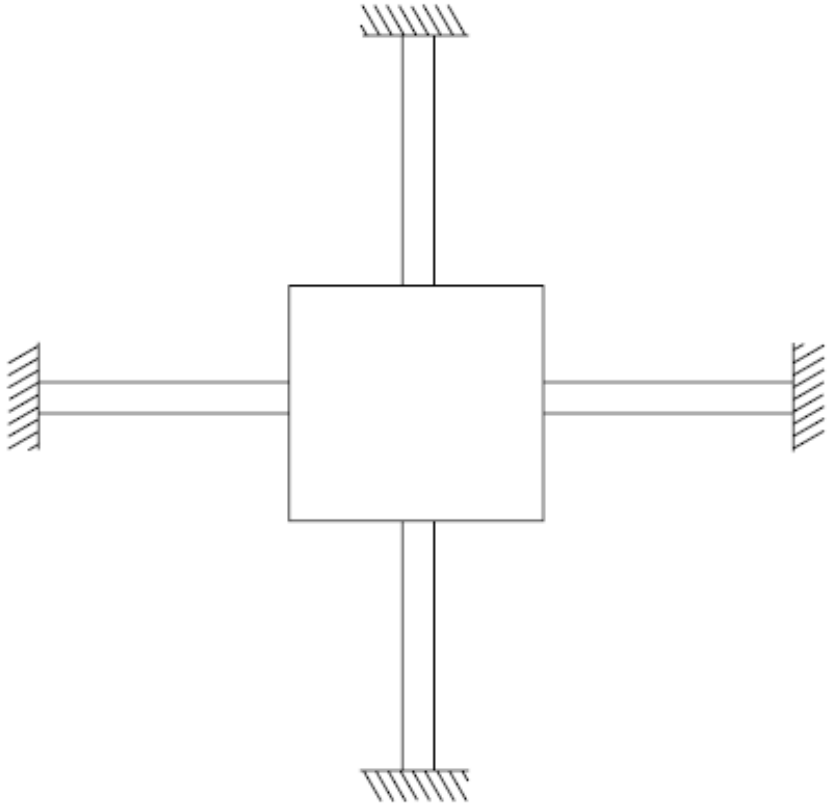
$$C = \varepsilon \frac{A}{d - \delta} = \frac{8.85 \times 10^{-12} \times (100 \times 10^{-6})^2}{(1 - 0.248) \times 10^{-6}} = 117.7 \times 10^{-15}\ \text{F}.$$

The relative change of the capacitance is

$$\frac{C - C_0}{C_0} \times 100\% = 33\%.$$

Problem 4: Design

A parallel-plate capacitor with four silicon support beams is shown below. The top plate has area of $1 \times 1\ \text{mm}^2$. The four support beams are each 500 μm long, 5 μm wide, and 0.3 μm in thickness. What is the force constant K_m experienced by the parallel-plate capacitor? (diagram not drawn to scale). The Young's modulus of silicon is 120 GPa.



1. 0.1 N/m
2. 0.4 N/m
3. 0.00013 N/m
4. 0.00052 N/m
5. 0.00003 N/m

Problem 1: Design

A parallel-plate capacitor has an area of $100 \times 100\,\mu m^2$. Calculate the capacitance values for two distances between the electrode plates: 1 and 0.5 μm. The medium is air.

Problem 2: Design

Two parallel capacitor plates have an area of 1 mm^2 and a spacing of 1 μm (air gap). One is fixed and another is suspended by a mechanical spring with a force constant of 1 N/m. What is the capacitance value (C_0) when no voltage is applied between the two plates?

1. $C_0 = 8.85 \times 10^{-12}$ f
2. $C_0 = 8.85 \times 10^{-9}$ f
3. $C_0 = 8.85 \times 10^{-6}$ f
4. $C_0 = 0$ f

Infrared Sensor

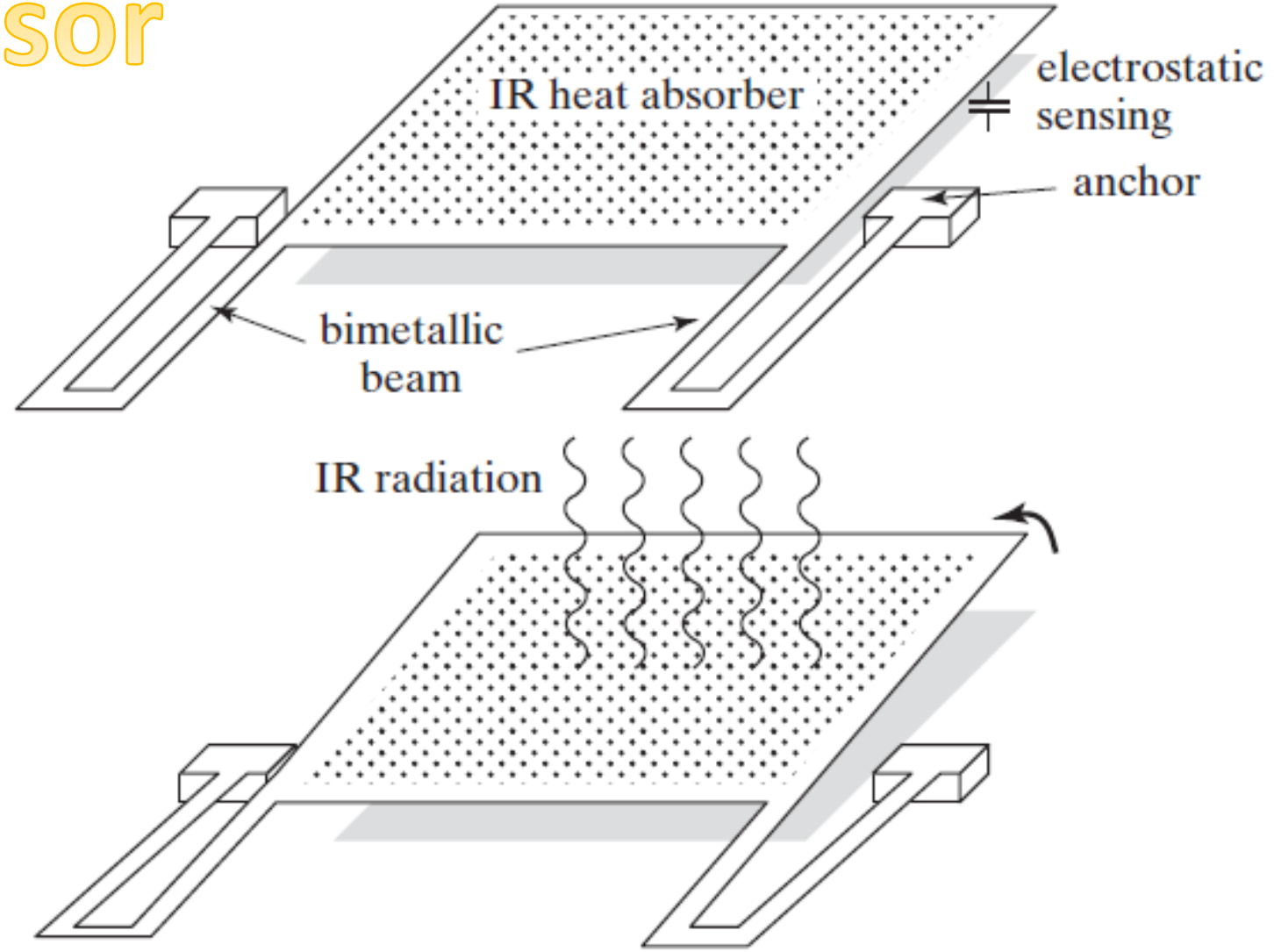


FIGURE 5.28

Infrared sensor based on capacitive sensing of the bending of thermal bimetallic beams under temperature changes due to absorbed IR power.